Integral Indefinida

Integrar es el proceso recíproco del de **derivar**, es decir, dada una función **f(x)**, se trata de buscaraquellas funciones **F(x)** que al ser derivadas conducen a **f(x)**.

Se dice, entonces, que **F(x) es una primitiva o antiderivada de f(x)**; dicho de otro modo las **primitivas de f(x)** son las **funciones derivables F(x)** tales que:

F'(x) = f(x).

Si una función f(x) tiene primitiva entonces tiene **infinitas primitivas**, diferenciándose todas ellas en una **constante**.

[F(x) + C]' = F'(x) + 0 = F'(x) = f(x)

Integral indefinida

Integral indefinida es el conjunto de las **infinitas primitivas** que puede tener una función.

Se representa por ∫ **f(x) dx**.

Se lee : **integral de x diferencial de x**.

∫ es el signo de integración.

f(x) es el **integrando** o función a integrar.

dx es **diferencial de x**, e indica cuál es la variable de la función que se integra.

C es la **constante de integración** y puede tomar cualquier valor numérico real.

Si F(x) es una **primitiva** de f(x) se tiene que:

∫ **f(x) dx = F(x) + C**

Para comprobar que la **primitiva** de una función es correcta basta con **derivar**.

Propiedades de la integral indefinida

1. Propiedad de linealidad: La i**ntegral de una suma** de funciones es igual a la **suma de las integrales** de esas funciones.

∫[f(x) + g(x)] dx =∫ f(x) dx +∫ g(x) dx

2. La **integral del producto de una constante** por una función es igual a la **constante por la integral** de la función.

∫ k f(x) dx = k ∫f(x) dx

Tabla de integrales

a, **k**, y **C** son constantes; **u** es una **función** y **u'** es la **derivada** de u.

$$\int dx = x + C$$

$$\int k\, dx = k \cdot x + C$$

$$\int u^n \cdot u'\, dx = \frac{u^{n+1}}{n+1} + C \qquad n \neq -1$$

$$\int \frac{u'}{u}\, dx = \ln u + C$$

$$\int a^u \cdot u'\, dx = \frac{a^u}{\ln a} + C$$

$$\int e^u \cdot u'\, dx = e^u + C$$

$$\int sen\, u \cdot u'\, dx = -\cos u + C$$

$$\int \cos u \cdot u'\, dx = \operatorname{sen} u + C$$

$$\int \frac{u'}{\cos^2 u}\, dx = \int \sec^2 u \cdot u'\, dx = \int \left(1 + tg^2 u\right) \cdot u'\, dx = tg\, u + C$$

$$\int \frac{u'}{sen^2 u} \cdot u'\, dx = \int cosec^2\, u \cdot u'\, dx = \int \left(1 + cotg^2 u\right) \cdot u'\, dx = -cotg\, u + C$$

$$\int \frac{u'}{\sqrt{1-u^2}}\, dx = arc\ sen\, u + C$$

$$\int \frac{u'}{1+u^2}\, dx = arc\ tg\, u + C$$

Si **u = x** (u' = 1), tenemos una **tabla de integrales** simples:

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$$

$$\int \frac{1}{x} dx = \ln x + C$$

$$\int a^x \, dx = \frac{a^x}{\ln a} + C$$

$$\int e^x \, dx = e^x + C$$

$$\int sen\, x \, dx = -\cos x + C$$

$$\int \cos x \, dx = \text{sen}\, x + C$$

$$\int \frac{1}{\cos^2 x} dx = \int \sec^2 x \, dx = \int \left(1 + tg^2 x\right) dx = tg\, x + C$$

$$\int \frac{1}{sen^2 x} \cdot dx = \int \text{cosec}^2 x \, dx = \int \left(1 + cotg^2 x\right) dx = -cotg\, x + C$$

$$\int \frac{1}{\sqrt{1 - x^2}} \cdot dx = arc\ sen\, x + C$$

$$\int \frac{1}{1 + x^2} \cdot dx = arc\ tg\, x + C$$

Integrales inmediatas

Integral de una constante

La **integral de una constante** es igual a la constante por x.

$$\int k\,dx = k\cdot x + C$$

Integral de cero

$$\int 0\,dx = C$$

Integral de una potencia

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + \mathrm{C}$$

$$\int u^n \cdot u'\,dx = \frac{u^{n+1}}{n+1} + \mathrm{C} \qquad n \neq -1$$

Ejemplos:

$$\int 7dx$$

$$\int 7dx = 7x + C$$

$$\int x^6 dx$$

$$\int x^6 dx = \frac{x^7}{7} + C$$

$$\int 7x^3 dx$$

$$\int 7x^3 dx = \frac{7x^4}{4} + C$$

$$\int x^{\frac{2}{3}} dx$$

$$\int x^{\frac{2}{3}} dx = \frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1} + C = \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + C = \frac{3x^{\frac{5}{3}}}{5} + C = \frac{3x\cdot\sqrt[3]{x^2}}{5} + C$$

$$\int \frac{3}{x^4} dx$$

$$\int \frac{3}{x^4}dx = \int 3x^{-4}dx = \frac{3x^{-4+1}}{-4+1} + C = \frac{3x^{-3}}{-3} + C = -x^{-3} + C = -\frac{1}{x^3} + C$$

$$\int \sqrt[3]{x}dx$$

$$\int \sqrt[3]{x}dx = \int x^{\frac{1}{3}}dx = \frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} + C = \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + C = \frac{3}{4}x^{\frac{4}{3}} + C = \frac{3}{4}x\sqrt[3]{x} + C$$

$$\int \frac{1}{\sqrt[4]{x}}dx$$

$$\int \frac{1}{\sqrt[4]{x}}dx = \int x^{\frac{-1}{4}}dx = \frac{x^{\frac{-1}{4}+1}}{\frac{-1}{4}+1} + C = \frac{4x^{\frac{3}{4}}}{3} + \mathrm{C} = \frac{4}{3}\sqrt[4]{x^3} + \mathrm{C}$$

$$\int \frac{1}{\sqrt[3]{x^2}}dx$$

$$\int \frac{1}{\sqrt[3]{x^2}}dx = \int x^{\frac{-2}{3}}dx = \frac{x^{\frac{-2}{3}+1}}{\frac{-2}{3}+1} + \mathrm{C} = \frac{x^{\frac{1}{3}}}{\frac{1}{3}} + \mathrm{C} = 3\sqrt[3]{x} + \mathrm{C}$$

$$\int \frac{1}{x^2\ \sqrt[5]{x^2}}dx$$

$$\int \frac{1}{x^2\ \sqrt[5]{x^2}}dx = \int x^{-2}\ x^{\frac{-2}{5}}dx = \int x^{\frac{-12}{5}}\ dx = \frac{x^{\frac{-12}{5}+1}}{\frac{-12}{5}+1} + \mathrm{C} =$$

$$= \frac{x^{\frac{-7}{5}}}{\frac{-7}{5}} + \mathrm{C} = -\frac{5}{7\sqrt[5]{x^7}} + \mathrm{C}$$

$$\int \left(x^4 - 6x^2 - 2x + 4\right)dx$$

$$\int \left(x^4 - 6x^2 - 2x + 4\right)dx = \frac{x^5}{5} - \frac{6x^3}{3} - x^2 + 4x + \mathrm{C}$$

$$\int \left(3\sqrt{x}+\frac{10}{x^6}\right)dx$$

$$\int \left(3\sqrt{x}+\frac{10}{x^6}\right)dx = \int \left(3x^{\frac{1}{2}}+10x^{-6}\right)dx = \frac{3x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+\frac{10x^{-6+1}}{-6+1}+C=$$

$$=\frac{3x^{\frac{3}{2}}}{\frac{3}{2}}-\frac{10}{5x^5}+C = 2x\sqrt{x}-\frac{2}{x^5}+C$$

$$\int \frac{x^2+\sqrt[3]{x^2}}{\sqrt{x}}dx$$

$$\int \frac{x^2+\sqrt[3]{x^2}}{\sqrt{x}}dx = \int \left(\frac{x^2}{\sqrt{x}}+\frac{\sqrt[3]{x^2}}{\sqrt{x}}\right)dx = \int \left(x^{\frac{3}{2}}+x^{\frac{1}{6}}\right)dx =$$

$$=\frac{x^{\frac{5}{2}}}{\frac{5}{2}}+\frac{x^{\frac{7}{6}}}{\frac{7}{6}} + C = \frac{2\sqrt{x^5}}{5}+\frac{6\sqrt[6]{x^7}}{7} + C = \frac{2x^2\sqrt{x}}{5}+\frac{6x\sqrt[6]{x}}{7} + C$$

$$\int \left(\sqrt{5x}+\sqrt{\frac{5}{x}}\right)dx$$

$$\int \left(\sqrt{5x}+\sqrt{\frac{5}{x}}\right)dx = \int \left(\sqrt{5}\cdot x^{\frac{1}{2}}+\sqrt{5}\cdot x^{-\frac{1}{2}}\right)dx = \sqrt{5}\cdot\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+\sqrt{5}\cdot\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+C=$$

$$\sqrt{5}\cdot\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+\sqrt{5}\cdot\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+C=\frac{2\sqrt{5}\cdot x\sqrt{x}}{3}+2\sqrt{5}\cdot\sqrt{x} = \frac{2x\sqrt{5x}}{3}+2\sqrt{5x}+C$$

$$\int \frac{3\sqrt{x}-5\sqrt[3]{x^2}}{2\sqrt[4]{x}}dx$$

$$\int \frac{3\sqrt{x}-5\sqrt[3]{x^2}}{2\sqrt[4]{x}}dx = \int \left(\frac{3\sqrt{x}}{2\sqrt[4]{x}}-\frac{5\sqrt[3]{x^2}}{2\sqrt[4]{x}}\right)dx = \int \left(\frac{3}{2}x^{\frac{1}{4}}-\frac{5}{2}x^{\frac{5}{12}}\right)dx =$$

$$= \frac{3}{2}\frac{x^{\frac{1}{4}+1}}{\frac{1}{4}+1} - \frac{5}{2}\frac{x^{\frac{5}{12}+1}}{\frac{5}{12}+1} + C = \frac{3}{2}\frac{x^{\frac{5}{4}}}{\frac{5}{4}} - \frac{5}{2}\frac{x^{\frac{17}{12}}}{\frac{17}{12}} + C =$$

$$= \frac{6}{5}\sqrt[4]{x^5} - \frac{30}{17}\sqrt[12]{x^{17}} + C$$

$$\int sen\, x \cos x \, dx$$

$$\int sen\, x \cos x \, dx = \frac{1}{2} sen^2 x + C$$

$$\int sen^2 \frac{x}{2} \cos \frac{x}{2} dx$$

$$\int sen^2 \frac{x}{2} \cos \frac{x}{2} dx = 2\int sen^2 \frac{x}{2} \cos \frac{x}{2} \cdot \frac{1}{2} dx = \frac{2}{3} sen^3 \left(\frac{x}{2}\right) + C$$

$$\int \left(\mathrm{tg}^3 x + \mathrm{tg}^5 x\right) dx$$

$$\int \left(\mathrm{tg}^3 x + \mathrm{tg}^5 x\right) dx = \int \mathrm{tg}^3 x \left(1 + \mathrm{tg}^2 x\right) dx = \frac{1}{4}\mathrm{tg}^4 x + C$$

$$\int \sec^2 x \sqrt{tgx}\, dx$$

$$\int \sec^2 x \sqrt{tgx}\, dx = \int \sec^2 x \,(tgx)^{\frac{1}{2}} dx = \frac{(tgx)^{\frac{3}{2}}}{\frac{3}{2}} + C =$$

$$= \frac{2}{3}\sqrt{tg^3 x} + C$$

$$\int \cot g\; x \sqrt{\ln senx}\, dx$$

$$= \int \frac{\cos x}{senx} (\ln senx)^{\frac{1}{2}} dx = \frac{(\ln senx)^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$= \frac{2}{3}\sqrt{(\ln senx)^3} + C$$

$$\int \frac{sen\,3x}{\sqrt{2+\cos 3x}}\,dx$$

$$\int \frac{sen\,3x}{\sqrt{2+\cos 3x}}\,dx = -\frac{1}{3}\int (2+\cos 3x)^{\frac{-1}{2}}\, sen\,3x\cdot(-3)\,dx =$$

$$= -\frac{2}{3}\sqrt{2+\cos 3x} \;+\; C$$

$$\int \left(\frac{\sec x}{1+tg\,x}\right)^2 dx$$

$$= \int \frac{\sec^2 x}{(1+tg\,x)^2}\,dx = \int \sec^2 x\;(1+tg\,x)^{-2}\,dx =$$

$$= -\frac{1}{1+tg\,x} \;+\; C$$

Integrales logaritmicas y exponenciales

$$\int \frac{1}{x}\,dx = \ln x + \mathrm{C}$$

$$\int \frac{u'}{u}\,dx = \ln u + \mathrm{C}$$

$$\int a^x\,dx = \frac{a^x}{\ln a} + \mathrm{C}$$

$$\int e^x\,dx = e^x + \mathrm{C}$$

$$\int a^u \cdot u'\,dx = \frac{a^u}{\ln a} + \mathrm{C}$$

$$\int e^u \cdot u'\,dx = e^u + \mathrm{C}$$

Ejemplos:

$$\int \frac{x^2}{x^3+8}\,dx$$

$$\int \frac{x^2}{x^3+8}dx = \frac{1}{3}\int \frac{3x^2}{x^3+8}dx = \frac{1}{3}\ln\left(x^3+8\right) + C$$

$$\int \cot g\ dx$$

$$\int \cot g\ dx = \int \frac{\cos\ x}{sen\ x}dx = \ln sen\ x + C$$

$$\int \frac{sen\ 2x}{1+sen^2 x}dx$$

$$\int \frac{sen\ 2x}{1+sen^2 x}dx = \int \frac{2\ sen\ x\ \cos x}{1+sen^2 x}dx = \ln\left(1+sen^2 x\right) + C$$

$$\int tg\ 5x\ dx$$

$$= \int \frac{sen\ 5x}{\cos 5x}dx = -\frac{1}{5}\ln\left(5x\right) + C$$

$$\int \frac{dx}{tg\ x}$$

$$\int \frac{dx}{tg\ x} = \int \frac{\cos x}{sen\ x}dx = \ln\left(senx\right) + C$$

$$\int \frac{1}{\sqrt{x}\left(1+\sqrt{x}\right)}dx$$

$$\int \frac{1}{\sqrt{x}\left(1+\sqrt{x}\right)}dx = 2\int \frac{\frac{1}{2\sqrt{x}}}{1+\sqrt{x}}dx = 2\ln\left(1+\sqrt{x}\right) + C$$

$$\int \frac{2x^3+x^2-x}{x^2}dx$$

$$\int \frac{2x^3+x^2-x}{x^2}dx = \int\left(2x+1-\frac{1}{x}\right)dx = x^2 - x - \ln x + \mathrm{C}$$

$$\int \frac{2^x}{3^x}dx$$

$$\int \frac{2^x}{3^x}\,dx = \int \left(\frac{2}{3}\right)^x dx = \frac{\left(\frac{2}{3}\right)^x}{\ln\left(\frac{2}{3}\right)} + C$$

$$\int x\, e^{x^2}\, dx$$

$$\int x\, e^{x^2}\, dx = \frac{1}{2}\int 2x\, e^{x^2}\, dx = \frac{1}{2}e^{x^2} + C$$

$$\int e^{sen^2 x}\, sen\, 2x\, dx$$

$$\int e^{sen^2 x}\, sen\, 2x\, dx = \int e^{sen^2 x}\, 2\, senx \cos x dx = e^{sen^2 x} + C$$

$$\int \frac{e^{\operatorname{tg} x}}{\cos^2 x}\,dx$$

$$\int \frac{e^{\operatorname{tg} x}}{\cos^2 x}\,dx = e^{\operatorname{tg} x} + C$$

$$\int \frac{5^{\sqrt{x}}}{x}\,dx$$

$$\int \frac{5^{\sqrt{x}}}{\sqrt{x}}\,dx = 2\int \frac{1}{2\sqrt{x}}\, 5^{\sqrt{x}}\,dx = \frac{2}{\ln 5}5^{\sqrt{x}} + C$$

$$\int \cos 5x\; e^{sen 5x}\,dx$$

$$\int \cos 5x\; e^{sen 5x}\,dx = \frac{1}{5}\int 5\cos 5x\; e^{sen 5x}\,dx = \frac{1}{5}e^{sen 5x} + C$$

$$\int \operatorname{cotg} x\, e^{\ln \operatorname{sen} x}\,dx$$

$$\int \frac{x}{x^2+2}\; 7^{\ln\left(x^2+2\right)}\,dx = \frac{1}{2}\int \frac{2x}{x^2+2}\; 7^{\ln\left(x^2+2\right)}\,dx = \frac{1}{2\ln 7}7^{\ln\left(x^2+2\right)} + C$$

$$\int \frac{e^{-2x}+e^{2x}}{2}\,dx$$

$$\int \frac{e^{-2x}+e^{2x}}{2}\,dx = \frac{1}{2}\left(-\frac{1}{2}\int e^{-2x}\cdot(-2)dx + \frac{1}{2}\int e^{2x}\cdot 2\,dx\right) =$$

$$= -\frac{1}{4}e^{-2x} + \frac{1}{4}e^{2x} + C$$

Integrales trigonométricas

$$\int sen\, x\; dx = -\cos x \; + C$$

$$\int sen\, u \cdot u'\, dx = -\cos u \; + C$$

$$\int \cos x \; dx = \text{sen}\; x + C$$

$$\int \cos u \; \cdot u'\, dx = \text{sen}\, u + C$$

$$\int \frac{1}{\cos^2 x} dx = \int \sec^2 x \; dx = \int \left(1 + tg^2 x\right) dx = tg\; x + C$$

$$\int \frac{u'}{\cos^2 u} dx = \int \sec^2 u \; \cdot u'\, dx = \int \left(1 + tg^2 u\right) \cdot u'\, dx = tg\; u + C$$

$$\int \frac{1}{sen^2 x} \cdot dx = \int cosec^2 x \; dx = \int \left(1 + cotg^2 x\right) dx = -cotg\; x + C$$

$$\int \frac{u'}{sen^2 u} \cdot u'\, dx = \int cosec^2 u \; \cdot u'\, dx = \int \left(1 + cotg^2 u\right) \cdot u'\, dx = -cotg\; u + C$$

Ejemplos:

$$\int \left(\cos x - senx\right) dx$$

$$\int \left(\cos x - senx\right) dx = senx + \cos x \; + C$$

$$\int \left(3x^2 - \sec^2 x\right) dx$$

$$\int \left(3x^2 - \sec^2 x\right) dx = x^3 + \text{tg}\, x$$

$$\int e^x \cos e^x \, dx$$

$$\int e^x \cos e^x \, dx = sen\, e^x \; + C$$

$$\int x\, \text{sen}\left(x^2 + 5\right) dx$$

$$\int x\,sen\left(x^2+5\right)dx=\frac{1}{2}\int sen\left(x^2+5\right)2x\,dx=-\frac{1}{2}\cos\left(x^2+5\right)\ +\ C$$

$$\int\frac{sen\left(\ln x\right)}{x}dx$$

$$\int\frac{sen\left(\ln x\right)}{x}dx=\int sen\left(\ln x\right)\frac{1}{x}dx=-\cos\left(\ln x\right)$$

$$\int\cos^3 x\,dx$$

$$\int\cos^3 x\,dx=\int\cos^2 x\ \cos x\,dx=\int\left(1-\operatorname{sen}^2 x\right)\cos x\,dx=$$

$$\int\left(\cos x-\operatorname{sen}^2 x\cos x\right)dx=\int\cos x\,dx-\int\operatorname{sen}^2 x\cos x\,dx=$$

$$\int\cos x\,dx-\frac{1}{3}\int 3\operatorname{sen}^2 x\cos x\,dx=\operatorname{sen} x-\frac{1}{3}\operatorname{sen}^3 x\ +\mathrm{C}$$

$$\int\operatorname{sen}^4 x\,dx$$

$$\int\operatorname{sen}^4 x\,dx=\int\left(\frac{1-\cos 2x}{2}\right)^2 dx=\int\frac{1-2\cos 2x+\cos^2 x}{4}dx=$$

$$=\frac{1}{4}\int dx-\frac{1}{4}\int 2\cos 2x\,dx+\frac{1}{4}\int\cos^2 x\,dx=$$

$$=\frac{1}{4}x-\frac{1}{4}\operatorname{sen} 2x+\frac{1}{4}\int\frac{1-\cos 4x}{2}dx=$$

$$=\frac{1}{4}x-\frac{1}{4}sen\ 2x+\frac{1}{8}x+\frac{1}{32}sen\ 4x+C$$

$$=\frac{3}{8}x-\frac{1}{4}sen\ 2x+\frac{1}{32}sen\ 4x+C$$

$$\int\operatorname{sen}^5 x\ \cos^2 x\,dx$$

$$\int sen^5 x\ \cos^2 x\,dx=\int sen\,x\ sen^4 x\ \cos^2 x\,dx=$$

$$=\int\left(1-\cos^2 x\right)^2 sen\,x\ \cos^2 x\,dx=$$

$$= \int \left(1 - 2\cos^2 x + \cos^4 x\right) sen\, x \ \cos^2 x dx$$

$$= \left(\int \cos^2 x \ sen\, x \ - 2\cos^4 x \ sen\, x + \cos^6 x\, sen\, x\right) dx$$

$$= -\frac{1}{3}\cos^3 x + \frac{2}{5}\cos^5 x - \frac{1}{7}\cos^7 x \ + \ C$$

$$\int \frac{dx}{\operatorname{sen} x \cos x}$$

$$= \int \frac{\operatorname{sen}^2 x + \cos^2 x}{\operatorname{sen} x \cos x} dx = \int \frac{\operatorname{sen} x}{\cos x} dx + \int \frac{\cos x}{\operatorname{sen} x} dx =$$

$$= -\ln(\cos x) + \ln(\operatorname{sen} x) \ + \mathrm{C} = \ln\left(\frac{\operatorname{sen} x}{\cos x}\right) \ + \mathrm{C} = \ln(\operatorname{tg} x) \ + \mathrm{C}$$

$$\int sen^2 4x\, dx$$

$$\int sen^2 4x\, dx = \int \frac{1 - \cos 8x}{2} dx = \frac{1}{2}x - \frac{1}{16} sen\, 8x \ + \ C$$

$$\int \cos^5 x \ dx$$

$$\int \cos^5 x \ dx = \int \cos^4 x \cos x\, dx = \int \left(1 - sen^2 x\right)^2 \cos x\, dx =$$

$$= \int \cos x\, dx - 2\int sen^2 x \ \cos x\, dx + \int sen^4 x \cos x\, dx =$$

$$= sen\, x - \frac{2}{3} sen^3 x + \frac{1}{5} sen^5 x \ + \ C$$

$$\int \sec^4 x\, dx$$

$$\int \sec^4 x\, dx = \int \sec^2 x \ \sec^2 x\, dx = \int \left(1 + tg^2 x\right) \sec^2 x\, dx =$$

$$\int \left(\sec^2 x + \sec^2 x \ tg^2 x\right) dx = tg\ x + \frac{1}{3} tg^3 x \ + \ C$$

$$\int \operatorname{tg}^2 x \ dx$$

$$\int \operatorname{tg}^2 x \ dx = \int \left(1 + \operatorname{tg}^2 x - 1\right) dx = \int \left(1 + \operatorname{tg}^2 x\right) dx - \int dx = \operatorname{tg} x - x + \mathrm{C}$$

$$\int \operatorname{cosec}^2(3x+1)\,dx$$

$$\int \operatorname{cosec}^2(3x+1)\,dx = \frac{1}{3}\int \operatorname{cosec}^2(3x+1)\,3\,dx = -\frac{1}{3}\operatorname{cotg}(3x+1) + C$$

$$\int \operatorname{cosec}^4 x\,dx$$

$$\int \operatorname{cosec}^4 x\,dx = \int \operatorname{cosec}^2 x\,\operatorname{cosec}^2 x\,dx =$$

$$\int \left(1+\operatorname{cotg}^2 x\right)\operatorname{cosec}^2 x\,dx = \int \operatorname{cosec}^2 x + \operatorname{cotg}^2 x\,\operatorname{cosec}^2 x\,dx =$$

$$= -\operatorname{cotg} x - \frac{1}{3}\operatorname{cotg}^3 x + C$$

$$\int \operatorname{cotg}^2 x\,dx$$

$$\int \operatorname{cotg}^2 x\,dx = \int \left[\left(1+\operatorname{cotg}^2 x\right)-1\right]dx = -\operatorname{cotg} x - x + C$$

Integrales trigonométricas inversas

$$\int \frac{1}{\sqrt{1-x^2}}\cdot dx = arc\ sen\,x + C$$

$$\int \frac{u'}{\sqrt{1-u^2}}\,dx = arc\ sen\,u + C$$

$$\int \frac{1}{1+x^2}\cdot dx = arc\ tg\,x + C$$

$$\int \frac{u'}{1+u^2}\,dx = arc\ tg\,u + C$$

Ejemplos:

$$\int \frac{x}{\sqrt{1-x^4}}\,dx$$

$$\int \frac{x}{\sqrt{1-x^4}}\,dx = \frac{1}{2}\int \frac{2x}{\sqrt{1-x^4}}\,dx = \frac{1}{2}\,\text{arc sen}\,x^2 + C$$

$$\int \frac{e^x}{\sqrt{1-e^{2x}}}dx$$

$$\int \frac{e^x}{\sqrt{1-e^{2x}}}dx = \int \frac{e^x}{\sqrt{1-\left(e^x\right)^2}}dx = arc\ sen\, e^x \ + C$$

$$\int \frac{1}{x\sqrt{1-\ln^2 x}}dx$$

$$\int \frac{1}{x\sqrt{1-\ln^2 x}}dx = \int \frac{1}{\sqrt{1-\ln^2 x}}\,\frac{1}{x}dx = arc\ sen\left(\ln x\right) \ + C$$

$$\int \frac{1}{\sqrt{x}\,\sqrt{1-x}}dx$$

$$\int \frac{1}{\sqrt{x}\,\sqrt{1-x}}dx = 2\int \frac{1}{\sqrt{1-\left(\sqrt{x}\right)^2}}\,\frac{1}{2\sqrt{x}}dx = 2\,arc\ sen\sqrt{x} \ + C$$

$$\int \frac{dx}{\sqrt{25-16x^2}}$$

$$\int \frac{dx}{\sqrt{25-16x^2}} = \int \frac{\frac{1}{5}}{\sqrt{1-\left(\frac{4}{5}x\right)^2}}dx = \frac{1}{4}\int \frac{\frac{4}{5}}{\sqrt{1-\left(\frac{4}{5}x\right)^2}}dx =$$

$$= \frac{1}{4}arc\ sen\left(\frac{4}{5}x\right) \ + C$$

$$\int \frac{1}{5+5x^2}dx$$

$$\int \frac{1}{5+5x^2}dx = \frac{1}{5}\int \frac{1}{1+x^2}dx = \frac{1}{5}arc\ tg\, x \ + C$$

$$\int \frac{1}{1+16x^2}dx$$

$$\int \frac{1}{1+16x^2}dx = \frac{1}{4}\int \frac{1}{1+\left(4x\right)^2}dx = \frac{1}{4}arc\ tg\, 4x \ + C$$

$$\int \frac{\cos x}{1+sen^2 x}\,dx$$

$$\int \frac{\cos x}{1+sen^2 x}\,dx = arc\ tg\left(senx\right)\ +\ C$$

$$\int \frac{x^2}{1+x^6}\,dx$$

$$\int \frac{x^2}{1+x^6}\,dx = \frac{1}{3}\int \frac{3x^2}{1+\left(x^3\right)^2}\,dx = \frac{1}{3}arc\ tgx^3\ +\ C$$

$$\int \frac{e^x}{1+e^{2x}}\,dx$$

$$\int \frac{e^x}{1+e^{2x}}\,dx = \int \frac{e^x}{1+\left(e^x\right)^2}\,dx = arc\ tg\, e^x\ +\ C$$

$$\int \frac{3}{1+9x^2}\,dx$$

$$\int \frac{3}{1+9x^2}\,dx = \int \frac{1}{1+\left(3x\right)^2}\cdot 3\,dx = arc\ tg\ 3x\ +\ C$$

$$\int \frac{1}{x^2+x+1}\,dx$$

Vamos a transformar el denominador de modo que podamos aplicar la fórmula de la integral del arcotangente.

Transformamos el denominador en un binomio al cuadrado.

$$\int \frac{1}{x^2+x+1}\,dx = \int \frac{1}{\left(x^2+x+\frac{1}{4}\right)-\frac{1}{4}+1}\,dx = \int \frac{1}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}\,dx =$$

Multiplicamos numerador y denominador por 4/3, para obtener uno en el denominador.

Dentro del binomio al cuadrado multiplicaremos por su raíz cuadrada de 4/3.

$$= \int \frac{\frac{4}{3}}{\left[\frac{2}{\sqrt{3}}\left(x+\frac{1}{2}\right)\right]^2+1}\,dx = \int \frac{\frac{2}{\sqrt{3}}\cdot\frac{2}{\sqrt{3}}}{\left[\frac{2}{\sqrt{3}}\left(x+\frac{1}{2}\right)\right]^2+1}\,dx =$$

$$= \frac{2}{\sqrt{3}} \int \frac{\frac{2}{\sqrt{3}}}{1 + \left(\frac{2}{\sqrt{3}} \frac{2x+1}{2} \right)^2} dx = \frac{2}{\sqrt{3}} \int \frac{\frac{2}{\sqrt{3}}}{1 + \left(\frac{2x+1}{\sqrt{3}} \right)^2} dx =$$

$$= \frac{2}{\sqrt{3}} \operatorname{arc\,tg} \frac{2x+1}{\sqrt{3}} + C$$